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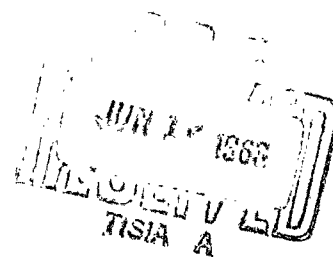CONTRACT AF 49(638)-255

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FINAL REPORT ON THEORETICAL AND
EXPERIMENTAL INVESTIGATIONS OF
THREE-DIMENSIONAL VISCOUS VORTEX FLOWS

by
Coleman duP. Donaldson
Roger D. Sullivan
Richard S. Snedeker

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Aeronautical Research Associates of Princeton, Inc.
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Final Report on Theoretical and Experimental
Investigations of Three-Dimensional
Viscous Vortex Flows

1. Introduction.

This report will present in summary the research
efforts carried out at ARAP during the past five years in
the field of viscous vortex motions. The work cited
constitutes a rather extensive program of theoretical and
experimental studies aimed at the better understanding of
vortex motions in general and of three-dimensional laminar
and turbulent incompressible driven vortices in particular.
The theoretical studies fall into three general categories;
the first and most important comprising velocity distribu-
tions for steady laminar motion, the second dealing with
unsteady velocity characteristics, and the third treating
temperature distributions for the steady motions first
considered. The experimental work, while intended primarily
to provide supporting evidence for conclusions based on the
analyses just mentioned, of necessity has had to include an
evaluation and discussion of the effects of turbulence in
real flows of the general type under consideration. (Prob-
lems associated specifically with turbulent vortices are
now under investigation at ARAP under a new AFOSR contract,
AF 49(638)-1187.) The ensuing discussion will present in
brief the work performed and the results and conclusions
reached in each of the areas of study already mentioned.
Full reporting of the details of this work is to be found
in separate documents for each part as follows:

Theoretical (C. duP. Donaldson and R. D. Sullivan)

> Examination of the Solutions of the Navier-Stokes
> Equations for a Class of Three-Dimensional Vortices:
>
> > Part I. Velocity Distributions for Steady Motion
> > AFOSR TN 60-1227.

Part II.   Velocity and Pressure Distributions for Unsteady Motion. ARAP Report No. 50.

Part III. Temperature Distributions for Steady Motion. ARAP Report No. 51.

Experimental (C. duP. Donaldson and R. S. Snedeker)

Experimental Investigation of the Structure of Vortices in Simple Cylindrical Vortex Chambers.

ARAP Report No. 47.

2. Theoretical Studies.

A. Velocity distributions for steady motion.

An extensive study was made of a class of solutions of the Navier-Stokes equations for which the velocity components are assumed to be of the form $u = u(r)$, $v = v(r)$, and $w = z\bar{w}(r)$. These solutions were found to represent a rather large class of three-dimensional viscous vortex motions. The class of solutions contains Burger's analytic solution for an unconstrained one-celled vortex as a special limiting case.

The solutions obtained show that vortex motions are possible which have more than one "cell". That is, the flow may not simply spiral in toward an axis and out along it as in a one-celled configuration but may have nested regions of successively reversed axial flow. The behavior of the solutions in passing from single to multiple-celled configurations was discussed and the solution for the extremely interesting case of a two-celled analogue to Burgers' unconstrained vortex, which probably occurs quite often in nature, was given in closed form.

An interesting outcome of the investigation which was discussed is that, for a given narrow range of dimensionless parameters governing the flow, no steady solutions of the Navier-Stokes equations of the type under investigation are possible.

B. Velocity and Pressure Distributions for Unsteady Motion.

The incompressible flow inside a porous cylindrical tube having steady radial inflow and which is rotated about its axis in a sinusoidal manner was obtained through solution of the complete Navier-Stokes equations. The nature of the tangential velocity profiles that result and the magnitudes of the mean pressure drop and the amplitude of the pressure fluctuations that are induced by the tangential velocities were discussed in some detail.

C. Temperature Distributions for Steady Motion.

The temperature distributions that occur inside a steadily rotated cylindrical vortex tube having steady radial inflow were obtained through solution of the Navier-Stokes equations. The nature of these temperature distributions in the case of both gases and liquids were discussed in some detail. The results obtained were compared with the analytical results given by Rott for the case when the velocities within the vortex are those given by Burgers' free vortex solution of the Navier-Stokes equations.

3. Experimental Studies.

Experiments were conducted in which the behavior of vortices in simple cylindrical vortex chambers was studied. The apparatus was designed so as to fulfill as nearly as possible the boundary conditions stipulated in the theoretical analysis. Special attention was given to the problem of the transition from one-celled to two-celled vortex structure. It was shown that for turbulent vortices in vortex chambers having length-to-diameter ratios in the range $0 < L/D < 5$, the character of the flow is primarily dependent on the ratio of a characteristic tangential velocity to a characteristic radial velocity V/U. In addition, it was shown that the Reynolds number based on the radial flow, while a parameter of major importance in laminar vortices,

plays but a minor role in turbulent vortex motion. Transition from one-celled to two-celled vortex structure started for all vortex configurations tested $(0 < L/D < 5)$ when V/U exceeded approximately 3.

4. Conclusions.

The studies of vortex flows discussed in the foregoing sections have helped to clarify a number of features of such flows which have been observed both in nature and in the many devices making use of certain properties of vortices. On the basis of the findings to date, it is felt that important and substantial progress has been made toward the fuller understanding of not only laminar vortex motions, both constrained and free, but of their turbulent counterparts as well.